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ABSTRACT

The mathematical growth of a pre-service elementary teacher over the period of a semester is detailed. This student had an unusual combination of very poor basic computational skills and a capacity for higher level problem solving involving algebraic notation. The extent to which one or two semesters of mathematics content, however well-structured, is sufficient to develop deep understanding, much less pedagogical content knowledge, particularly when growth of mathematical reasoning and understanding are coupled with incorrect or inappropriate algorithms which have become automated over several years of use is detailed. The paper provides evidence that this student acquired new insights and understandings about mathematics and experienced changes in her attitudes and beliefs. However, it is not known whether the changes in attitudes and beliefs are short- or long-term nor whether what she learned is truly "owned" by her. (Contains 13 references.) (Author/MM)

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GROWTH AND DEVELOPMENT OF PRE-SERVICE ELEMENTARY TEACHERS' MATHEMATICAL KNOWLEDGE[1]

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We detail the mathematical growth of a pre-service elementary teacher of the period of a semester. This student had an unusual combination of very poor basic computational skills and a capacity for higher level problem solving involving algebraic notation. We discuss the extent to which one or two semesters of mathematics content, however well-structured, is sufficient to develop deep understanding, much less pedagogical content knowledge, particularly when growth of mathematical reasoning and understanding are coupled with incorrect or inappropriate algorithms which have become automated over several years of use. We provide evidence that this student acquired new insights and understandings about mathematics and experienced changes in her attitudes and beliefs. However, we do not know whether the changes in attitudes and beliefs are short- or long-term nor whether what she learned is truly "owned" by her.

Introduction

We are concerned, in this study, with the development of deep understanding of mathematics by pre-service elementary teachers. Those who teach mathematics content courses for pre-service elementary teachers are aware that, as they begin the mathematics content course, pre-service teachers' attitudes to mathematics are generally instrumental, focused on formulas and correct answers. We discuss and analyze what constitutes evidence of the development of deep understanding of mathematics by focusing on a young woman enrolled in a mathematics content course for pre-service teachers at a community college. This case study was carried out in a mathematics classroom and has important implications for linking research and practice. The issues raised in this study focus on what is theoretically desirable in a content course for pre-service teachers versus what might be practically attainable in one or two semesters. Effecting change in classroom practice based on research is not always straight-forward (McGowen & Vrooman, 1998). Yet research that provides evidence for what student growth in understanding and competence is attainable, and what the limits and applicability of those attainments are, is vital if mathematics content courses are to impact pre-service teachers deeply.

Many authors have addressed the question of growth of knowledge and growth of understanding in mathematics (for example: Ball & Bass, 2000; Clarke, Helme & Kessel, 1996; Davis & McGowen, 2001; Hiebert & Carpenter, 1992; Ma, 1999; McGowen, & Davis, 2001; Mewborn,1999; Pirie & Kieren, 1994; Simon & Tzur, 1999; Tall, 1995). The issues discussed in the literature acknowledge the importance and desirability of the growth of conceptual understanding and technical competence. What is largely not addressed, however, is the extent and degree to which desired growth can be supported and encouraged in one or two semesters, and to what extent such a limited exposure to mathematics content for pre-service elementary teachers can impinge on decades of relatively negative experiences.

From a more quantitative angle, Hake (1998) and McGowen & Davis (2001) have utilized a measure of relative increase, pre-test to post-test, for whole classes and for individual students. This statistic—*gain factor*—is calculated as the ratio $gain = (post\text{-}test\% - pre\text{-}test\%)/(100\% - pre\text{-}test\%)$. Indications are that this statistic is related to student growth in a more cognitive sense (Hake, 1998; McGowen & Davis, 2001).

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The main purpose of this study is not simply to re-iterate that pre-service teachers generally consider mathematics to consist of formulas and correct answers. It is also about how a particular person—Holly—develops considerably deeper understanding of mathematics over a semester, and how Holly's knowledge of mathematics may yet be insufficient for her to teach elementary mathematics effectively.

Methods

The principal question of this study—"how does a pre-service teacher develop a deeper understanding of mathematics over a semester?"—is not easy to answer, and we need to indicate how our modes of inquiry will do that, and how our theoretical framework supports our question. At Harper College, prospective elementary teachers are required to pass a pencil-and-paper competency exam of basic arithmetic computations with a grade of 80% as a condition of successful completion of the first mathematics content course of a two course sequence. Students take this exam during the first week of the semester. They have two additional opportunities to take and pass the exam during the semester. Written work in the form of in-class tests, out of class assignments, and student reflections was also collected from students. Holly's written work was examined and analyzed in detail. Holly's work was analyzed in detail because of extraordinary incongruities in poor basic computational skills and outstanding higher order thinking skills. The individual gain factors were calculated for all students and Holly's gain factor located in the distribution for the class as a whole. Holly was also interviewed in relation to both her in-class and out-of-class written work.

Results

On her initial attempt the first week of class, Holly correctly answered six of thirty questions (20%). Her answer to the first of the following two problems is shown in Figure 1. She did not attempt the second.

(a) $3\frac{1}{2} \times 2\frac{8}{15}$ (b) $1\frac{3}{4} \div \frac{1}{2}$

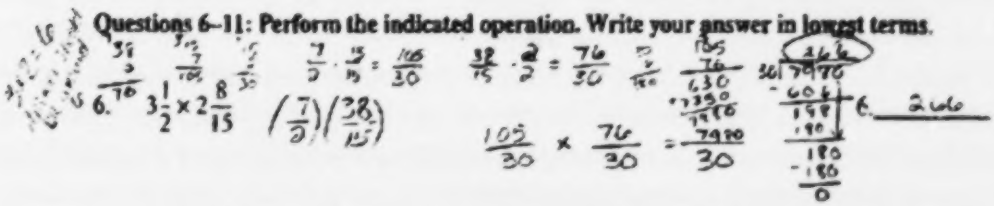
Questions 6–11: Perform the indicated operation. Write your answer in lowest terms.
6. $3\frac{1}{2} \times 2\frac{8}{15}$
$\frac{105}{30} \times \frac{76}{30} = \frac{7980}{30}$
6. 266

FIGURE 1. *Holly's response to question on initial competency exam (Week 1)*

Only nine of the forty-six students who took the competency exam the first week of the Fall 2001 semester gave a correct response to problem (a). Thirty-two students—including Holly—gave incorrect responses to this question. Five students made no attempt to answer the question. Fourteen different incorrect responses were given for this problem, including Holly's answer of 266. Fifteen different incorrect responses were given for the second problem.

On her second attempt eight weeks later Holly, like many others in the course, more than doubled her initial score to 50%. For Holly and the other 28 of 46 students who scored less than 30% initially, doubling the initial score on the second attempt midway through the semester indicates significant improvement but does not come close to satisfying the department's required passing score of 80%. The work on her 2nd attempt of the competency exam Question 6 (multiplication of two mixed numbers) is difficult to explain (Figure 2). Her work for Question 17 does not justify the answer and we are left wondering how it was actually determined. Though her work on Questions 27 and

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28 is procedural and not as efficient as one might hope, she has represented these two problems appropriately and has written correct answers to these questions which she had not even attempted eight weeks earlier.

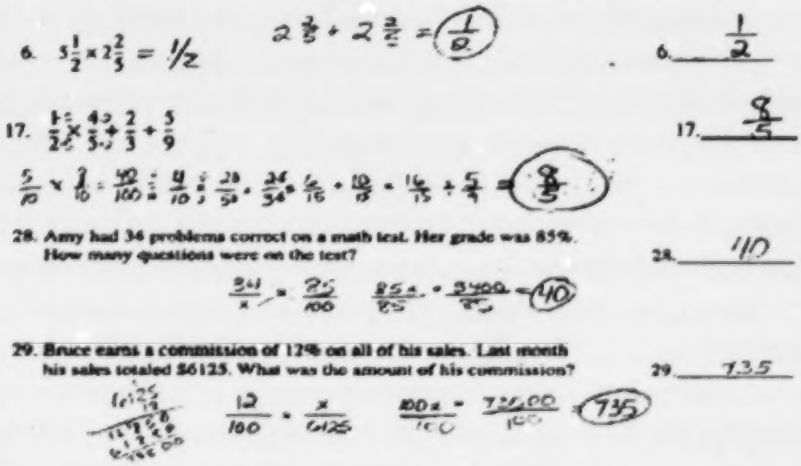
6. $5\frac{1}{2} \times 2\frac{2}{5} = 1/2$
$2\frac{3}{5} + 2\frac{2}{5} = \frac{1}{0}$
6. $\frac{1}{2}$
17. $\frac{1}{2} \times \frac{4}{5} + \frac{2}{3} + \frac{5}{9}$
17. $\frac{8}{5}$
$\frac{5}{10} \times \frac{2}{10} = \frac{10}{100} = \frac{4}{10} = \frac{28}{56} \cdot \frac{24}{54} \cdot \frac{16}{16} \cdot \frac{10}{15} \cdot \frac{16}{15} + \frac{5}{9} = \frac{8}{5}$
28. Amy had 34 problems correct on a math test. Her grade was 85%. How many questions were on the test?
28. 40
$\frac{34}{x} = \frac{85}{100}$ $\frac{85x}{85} = \frac{3400}{85} = 40$
29. Bruce earns a commission of 12% on all of his sales. Last month his sales totaled $6125. What was the amount of his commission?
29. 735
$\frac{12}{100} = \frac{x}{6125}$ $\frac{100x}{100} = \frac{73500}{100} = 735$

FIGURE 2. Responses from Holly's second attempt on the departmental competency exam (Week 9).

In an interview after taking the competency exam for the second time, Holly mentioned that she had studied the textbook's explanation of the theorem of addition of fractions with unlike denominators and the definition of multiplication of fractions to figure out how to do problems like Question 17, above. Holly had been unable to attend any of several review sessions due to scheduling conflicts so she attempted to learn a procedure she knew she did not know on her own using her textbook as reference.

In her mathematical autobiography, written at the beginning of the 16 week course, Holly describes her previous experiences with mathematics as follows:

> "Out of all the subjects that I have had to endure over the course of my education, Math has always been the most difficult for me to understand....The basis of all my math problems stems from the fact that I never learned the basics in mathematics....If I am asked a simple question such as: What does six times seven equal? I literally have to stop and think about it for a minute before I can come up with an answer."

Holly's responses to questions on an exam taken during the 12th week of the semester provide a startling contrast to her work on the initial basic skills exam and her midterm second attempt on this exam.

Question 5:

Consider the set of numbers A = {5, 6, 7, 8} with a made-up operation called *.
The rules for * are shown in the table below.

a) Is * commutative for set A? Justify your response.

*	5	6	7	8
5	8	7	6	5
6	7	6	5	8
7	6	5	8	7
8	5	8	7	6

FIGURE 3. *Operation table for question 5.*

Student response:

Yes. * is commutative.

5 * 5 = 8	6 * 5 = 7	7 * 5 = 6	8 * 5 = 5
5 * 6 = 7	6 * 6 = 6	7 * 6 = 5	8 * 6 = 8
5 * 7 = 6	6 * 7 = 5	7 * 7 = 8	8 * 7 = 7
5 * 8 = 5	6 * 8 = 8	7 * 8 = 7	8 * 8 = 6

5 * 6 and 6 * 5 both have the same result of 7.
7 * 5 and 5 * 7 both have the same result of 6.
5 * 8 and 8 * 5 both have the same result of 5.
6 * 7 and 7 * 6 both have the same result of 5.
6 * 8 and 8 * 6 both have the same result of 8.
8 * 7 and 7 * 8 both have the same result of 7.

Therefore, the set A is commutative since this is true for all.

Recall that the *gain factor* for an individual student is the ratio (*post-test%* – *pre-test%*)/(100% – *pre-test%*). Classes with average high gain factor (≥ 0.48) correspond generally to reform type classes rather than more traditional chalk and talk type or drill type classes (Hake, 1998). Further, individual student growth in understanding of mathematics, as well as basic skills improvement, seems to be connected with higher individual gain factors (McGowen & Davis, 2001). Holly's individual gain factor was the 5th highest in a class of 33 students, and was a striking 0.85 compared with a class average of 0.58. From this perspective alone we should be alerted to possible significant cognitive growth in Holly's understanding of mathematics.

Holly's final grade (87%) was the sixth highest of the thirty-three students who took the final exam. Her final self-evaluation submitted with her portfolio in the sixteenth week provides confirmation of the growth of her mathematical understanding, and documents her changed attitudes and beliefs about mathematics and what it means to learn mathematics:

> "Reflecting on … mathematics in August versus today now almost sixteen weeks later there is a significant difference on the way that I personally approach a math problem. In August I would: do the computation, focus of my attention being on the numbers within the problem, and usually not considering the content of the problem at hand. Once I received what I thought was the correct answer I would move on to the next problem and start the whole process over again. Now in December I focus my attention on the entire problem not just the numbers within it, I now look for alternative strategies and select the most appropriate and consider the context of the problem. Once I get a result I go through a series of reality checks: is this answer reasonable? Does this answer make sense in the context of the problem? Then I interpret my result by reflecting upon it and consider are there any other answers, strategies, possible explanations for the problem? When this process is complete only then move and I then on to the next problem to repeat the steps I just mentioned."

Holly shows evidence of higher order thinking and algebraic skills, which is particularly puzzling in light of her inability to carry out basic arithmetic computations. Evidence of her ability to think a problem through and reflect on her answer conceptually comes from her work on prime number values of a polynomial. Holly was interviewed after the Second Unit Exam had been submitted and asked to explain how the answer to the question below was determined. She said that both she and her partner had experienced an "AHHHH" moment. When asked to explain exactly what they had noticed that produced that AHHHH moment, Holly pulled out a copy of the spreadsheet she had created for testing various values of n in the equation and provided the following explanation:

Proof and Justification was another method that was learned and relearned over the semester. In a mathematical problem there lays a claim setting restrictions on the problem's possible solutions and which must abide by these restrictions to be labeled as a correct solution to the problem. Example six on the group exam gives such an example of claims and their relation to the process of Proof and Justification: Does the expression $n \wedge 2 - n + 41$ give only prime numbers? *Explain your reasoning and justify your answer.*

> "No, this formula does not always generate a prime number. If $n = 41$, the square root of the sum is 41. $41^2 - 41 + 41 = 1681$ and $1681 = 41^2$
>
> Our first approach to this problem was actually computing the expression and plugging in numbers up to 108 for n to determine if the result was a prime number. Next we looked at the actual number 41 more closely and tried other prime numbers in its place in the expression above to see if it would also generate prime numbers.
>
> This did not occur when we replaced 41 with any other prime number, therefore we knew that there was something special about the number 41 in this particular expression. Having this information on the number 41 created an AHHHHH moment in our minds and we decided to plug in 41 for n to see if the expression worked. This proved false and we found an example that proved that this expression was not true for all.
>
> In this problem the claim that existed was the number 41 giving only prime numbers, in other words is this prime number true for all numbers that you plug in the expression. To disprove a claim that there exists only prime numbers give one counter example, and now the claim true for all is proved false and is justified as well."

Holly's ability to reason mathematically by week 12 of the semester—to appropriately and effectively argue using a proof by exhaustion, to make and test conjectures, to recognize and represent relationships symbolically, and to clearly articulate what was done, both verbally and in writing—to make sense of the mathematics—provides a startling contrast to her earlier work on the departmental competency exam. Further evidence for this is provided by her responses to Question 11 on the same exam (Figure 4):

11. Find the smallest counting number that has the following properties: When it is divided by 2, the result is a perfect square number, and when it is divided by 3, the result is a perfect cube. Document all efforts and justify your conclusion.

"n" is divisible by 6, therefore "a" and "b" must be multiple of 6.
and $n = b^2 \times 2$
$n = a^3 \times 3$
Since $a^3 \times 3 = n$ will have the smaller multiple of 6, we started by using 6.
$6^3 \times 3 = 216 \times 3 = 648$
Then divided 648 by 2 - $648 \div 2 = 324$, took the square root $= 18$.

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FIGURE 4. Question 11: Unit II Exam (week 12)

Student response:

... even [is written next to the phrase *divided by 2*] and $n = b^2 \times 2$; $n = a^3 \times 3$.

[The number] must be divisible by 6. Therefore "a" and "b" must be multiples of 6.

Since $a^3 \times 3 = n$ will have the smaller multiples of 6, we started by using 6.

$$6^3 \times 3 = 216 \times 3 = 648$$

Then divided 648 by 2: 648/2 = 324, took the square root = 18.

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Holly's work on the open response final exam provides evidence of her growing ability to think flexibly and of her improved competence with fractions acquired during the sixteen weeks:

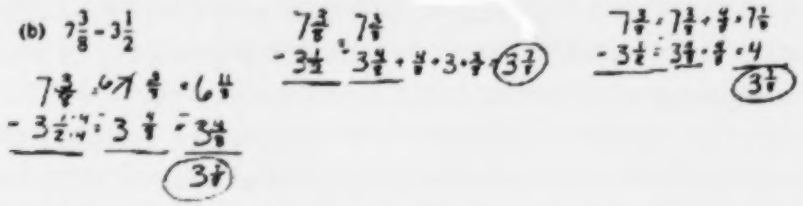

FIGURE 5. Holly's work on fraction problems in the final examination

The week before the final exam, Holly still had not passed the competency exam with a grade of 80%—she had a 60%. Yet, a week later, on the final exam, she was able to not only do computations correctly with similar fraction problems, but demonstrated her ability to do the calculations in a variety of ways. She was given an Incomplete as her course grade and allowed to enroll in the subsequent course subject to passing the departmental final within six weeks of the next semester start-up. She did so and passed the competency exam with a score of 83%.

Analysis

How does our data illuminate the issue of the growth of deeper understanding of mathematics by pre-service elementary teachers? Being able to use various algorithms correctly and selecting an efficient or elegant method of solution are often considered indicators of growth. For us, indicators of growth in deeper understanding of mathematics means that students are thinking more flexibly and systematically, able to articulate what was done, justifying why a method was used, generalizing their results, recognizing and building on connections between problems in different contexts and between various mathematics topics, and recognizing the role of definitions and proofs. Holly's work on the competency exam contrasted with her classwork and exams during the semester challenges us to re-examine our traditional notions of evaluation and assessment. What constitutes evidence of learning? What do our traditional methods of assessment tell us?

How much of what she has learned will Holly retain and use over time? On the one hand, Holly, like many of her colleagues, has struggled to re-learn computational algorithms which had become deeply-fixed habits of automatic, incorrect responses learned previously, and she experiences debilitating mathematics and test anxiety under stressful conditions like the high-stakes competency exam. Despite these obstacles, Holly, and many others, demonstrate the ability to develop more flexible ways of thinking as they investigate problems and reflect on their work during the semester. They make sense of the mathematics they are learning and build connections—having realized that in order to make connections, they "have to have something to connect." Their language grows more precise, what they focus attention on initially changes, and mathematical marks become truly symbolic. The change in attitudes and beliefs, coupled with their growth in understanding and improved competency is evidence of the changes that have taken place.

The inability of preservice teachers like Holly to demonstrate competency of basic arithmetic computations at the beginning of the semester has not proven to be an accurate predictor of their ability to think mathematically—to represent problem situations, make connections, justify their work, use mathematical arguments, or to think flexibly and

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systematically. Since Fall 1996, only 4 of 206 pre-service elementary teachers who attempted the course passed the departmental competency exam on their initial attempt with a grade of 80% or higher of those who enrolled in the first content course. One hundred fifty-five of the 206 students completed the course and took the final exam—a 75% completion rate. Of those who completed the course, 96% (149 of 155) were successful, i.e., passed the course with a grade of C or better and passed the departmental competency exam with a score of 80% or better on either their second or third attempt.

Discussion

What are reasonable expectations of students who enroll in a mathematics content course with deeply held beliefs about mathematics and what it means to learn mathematics that reflect a strictly utilitarian perspective of mathematics that often limits their mathematical vision? The complexity of the task in determining answers to our questions is illustrated by an examination of the work and comments of one elementary pre-service student. If, as seems clear to us, deeper understanding develops over time, should we focus our efforts on providing students with pedagogically sound experiences of what it means to learn mathematics and then direct subsequent efforts to building on and transforming that foundation knowledge of mathematics into pedagogically useful content knowledge? Can students like Holly think about how to use their mathematical knowledge in the classroom when their mathematical foundation lacks both conceptual understanding and skill competency in the mathematics that is the basis for what they will be teaching? Holly's initiative in opening and following the textbook to carry out fraction calculations is to be applauded—she wants to be a teacher and is highly motivated. However, her interpretation of the text is an example of "a little learning is a dangerous thing." What is symbolic and meaningful for those with the mathematical knowledge to interpret the symbols and an understanding of the implicit assumptions about the product in the denominator is, in the given context, simply a rule to follow in order to get the answer for students like Holly who attempt to interpret the marks with limited mathematical understanding.

What constitutes evidence of deep understanding? Is it being able to recognize which algorithm a student is using? Is it being able to calculate an answer using various algorithms and being able to choose between known alternative methods and selecting the most efficient/elegant method of solution? Is it being able to articulate what was done, why the particular method used is valid? Is it demonstrating the ability to recognize connections given isomorphic problems in different contexts? Is profound understanding attainable in a sixteen week semester? Over two semesters? By the time the pre-service teacher graduates? If a deep understanding of the mathematics that pre-service teachers need to know takes years to develop, then we need to consider what is possible over time and what is reasonable to expect of pre-service teachers.

The pre-service elementary students who enroll in mathematics content courses at many institutions — particularly at two-year colleges and other institutions with open enrolment policies—are in need of a mathematics content foundation to think with before they are able to think about how to use their mathematical knowledge in the classroom. If our goal is the development of deep understanding of mathematics by those who will be our future teachers, then the various constituencies involved in teacher preparation need to align their instructional goals and practices across institutions so that the development of profound understanding can develop over time. There is, in our view, no doubt that this is a developmental issue, and one that is compounded because it is about development in mathematics by adults for a professional purpose. It seems to us that there are profoundly important educational issues at stake

here, and that much focused and penetrating research is required to answer a number of fundamental questions. We believe the results presented here are of significance to those interested in the growth of mathematical knowledge of pre-service elementary teachers. Above all else, we feel, this is a question of considerable interest for the mathematics education community.

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